Chapter 2

2.1

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \tag{1}$$

Energy balance:

$$C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} = w_1C(T_1 - T_{ref}) + w_2C(T_2 - T_{ref}) - w_3C\left(T_3 - T_{ref}\right) \tag{2}$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \tag{3}$$

b) From Eq. 2, substituting Eq. 3

$$\rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} = w_1C(T_1 - T_{ref}) + w_2C(T_2 - T_{ref}) - \left(w_1 + w_2\right)C\left(T_3 - T_{ref}\right) \tag{4}$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1T_1 + w_2T_2 - (w_1 + w_2)T_3 \tag{5}$$

The simplified model now consists only of Eq. 5.

Degrees of freedom for the simplified model:

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$w_1 = w_1(t)$
$w_2 = w_2(t)$
$T_1 = T_1(t)$
$T_2 = T_2(t)$

Thus, N_F is reduced to 0.

2.2

Energy balance:

$$C_p \frac{d\left[\rho V(T - T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho VC_p \frac{dT}{dt} = wC_pT_i - wC_pT - UA_s(T - T_a) + Q$$
$$\rho VC_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

b) T increases if T_i increases and vice versa.

T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T-T_a)$, the contents are heated, and $T > T_i$.

2.3

a) Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \tag{1}$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \tag{2}$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

Let P_a be the ambient pressure.

Then
$$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \tag{3}$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \tag{4}$$

b) Seven parameters: ρ, A_1, A_2, g, g_c, R_2, R_3

Five variables : h_1, h_2, w_1, w_2, w_3

Four equations

Thus $N_F = 5 - 4 = 1$

1 input = w_1 (specified function of time)
4 outputs = h_1, h_2, w_2, w_3

2.4

Assume constant liquid density, ρ . The mass balance for the tank is

$$\frac{d(\rho Ah + m_g)}{dt} = \rho(q_i - q)$$

Because ρ, A, and m_g are constant, this equation becomes

$$A\frac{dh}{dt} = q_i - q \tag{1}$$

The square-root relationship for flow through the control valve is

$$q = C_v\left(P_g + \frac{\rho gh}{g_c} - P_a\right)^{1/2} \tag{2}$$

From the ideal gas law,

$$P_g = \frac{(m_g / M)RT}{A(H-h)} \tag{3}$$

where T is the absolute temperature of the gas.

Equation 1 gives the unsteady-state model upon substitution of q from Eq. 2 and of P_g from Eq. 3:

$$A\frac{dh}{dt} = q_i - C_v\left[\frac{(m_g / M)RT}{A(H-h)} + \frac{\rho gh}{g_c} - P_a\right]^{1/2} \tag{4}$$

Because the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and Eq. 2 shows that the system is independent of P_a.

2.5

a) For linear valve flow characteristics,

$$w_a = \frac{P_d - P_1}{R_a}, \quad w_b = \frac{P_1 - P_2}{R_b}, \quad w_c = \frac{P_2 - P_f}{R_c} \tag{1}$$

Mass balances for the surge tanks

$$\frac{dm_1}{dt} = w_a - w_b, \qquad \frac{dm_2}{dt} = w_b - w_c \tag{2}$$

where m_1 and m_2 are the masses of gas in surge tanks 1 and 2, respectively.

If the ideal gas law holds, then

$$P_1 V_1 = \frac{m_1}{M} R T_1, \qquad P_2 V_2 = \frac{m_2}{M} R T_2 \tag{3}$$

where M is the molecular weight of the gas
T_1 and T_2 are the temperatures in the surge tanks.

Substituting for m_1 and m_2 from Eq. 3 into Eq. 2, and noticing that V_1, T_1, V_2, and T_2 are constant,

$$\frac{V_1 M}{R T_1}\frac{dP_1}{dt} = w_a - w_b \quad \text{and} \quad \frac{V_2 M}{R T_2}\frac{dP_2}{dt} = w_b - w_c \tag{4}$$

The dynamic model consists of Eqs. 1 and 4.

b) For adiabatic operation, Eq. 3 is replaced by

$$P_1\left(\frac{V_1}{m_1}\right)^{\gamma} = P_2\left(\frac{V_2}{m_2}\right)^{\gamma} = C, \text{ a constant} \tag{5}$$

or

$$m_1 = \left(\frac{P_1 V_1^{\gamma}}{C}\right)^{1/\gamma} \quad \text{and} \quad m_2 = \left(\frac{P_2 V_2^{\gamma}}{C}\right)^{1/\gamma} \tag{6}$$

Substituting Eq. 6 into Eq. 2 gives,

$$\frac{1}{\gamma}\left(\frac{V_1^{\gamma}}{C}\right)^{1/\gamma} P_1^{(1-\gamma)/\gamma}\frac{dP_1}{dt} = w_a - w_b$$

$$\frac{1}{\gamma}\left(\frac{V_2^{\ \gamma}}{C}\right)^{1/\gamma} P_2^{\ (1-\gamma)/\gamma}\frac{dP_2}{dt} = w_b - w_c$$

as the new dynamic model. If the ideal gas law were not valid, one would use an appropriate equation of state instead of Eq. 3.

2.6

a) Assumptions:

1. Each compartment is perfectly mixed.
2. ρ and C are constant.
3. No heat losses to ambient.

Compartment 1:
Overall balance (No accumulation of mass):

$$0 = \rho q - \rho q_1 \qquad \text{thus} \qquad q_1 = q \tag{1}$$

Energy balance (No change in volume):

$$V_1\rho C\frac{dT_1}{dt} = \rho qC(T_i - T_1) - UA(T_1 - T_2) \tag{2}$$

Compartment 2:

Overall balance:

$$0 = \rho q_1 - \rho q_2 \qquad \text{thus} \qquad q_2 = q_1 = q \tag{3}$$

Energy balance:

$$V_2\rho C\frac{dT_2}{dt} = \rho qC(T_1 - T_2) + UA(T_1 - T_2) - U_cA_c(T_2 - T_c) \tag{4}$$

b) Eight parameters: ρ, V_1, V_2, C, U, A, U_c, A_c
Five variables: T_i, T_1, T_2, q, T_c
Two equations: (2) and (4)

Thus $N_F = 5 - 2 = 3$

2 outputs = T_1, T_2
3 inputs = T_i, T_c, q (specify as functions of t)

c) Three new variables: c_i, c_1, c_2 (concentration of species A).
Two new equations: Component material balances on each compartment.
c_1 and c_2 are new outputs. c_i must be a known function of time.

2.7

As in Section 2.4.2, there are two equations for this system:

$$\frac{dV}{dt} = \frac{1}{\rho}(w_i - w)$$
$$\frac{dT}{dt} = \frac{w_i}{V\rho}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

Results:

(a) Since w is determined by hydrostatic forces, we can substitute for this variable in terms of the tank volume as in Section 2.4.5 case 3.

$$\frac{dV}{dt} = \frac{1}{\rho}\left(w_i - C_v\sqrt{\frac{V}{A}}\right)$$
$$\frac{dT}{dt} = \frac{w_i}{\rho V}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

This leaves us with the following:

5 variables: V, T, w_i, T_i, Q
4 parameters: C, ρ, C_v, A
2 equations

The degrees of freedom are $5 - 2 = 3$. To make sure the system is specified, we have:

2 output variables: T, V

2 manipulated variables: Q, w_i

1 disturbance variable: T_i

(b) In this part, two controllers have been added to the system. Each controller provides an additional equation. Also, the flow out of the tank is now a manipulated variable being adjusted by the controller. So, we have

4 parameters: C, ρ, T_{sp}, V_{sp}

6 variables: V, T, w_i, T_i, Q, w

4 equations

The degrees of freedom are $6-4=2$. To specify the two degrees of freedom, we set the variables as follows:

2 output variables: T, V

2 manipulated variables (determined by controller equations): Q, w

2 disturbance variables: T_i, w_i

2.8

Additional assumptions:

(i) Density of the liquid, ρ, and density of the coolant, ρ_J, are constant.
(ii) Specific heat of the liquid, C, and of the coolant, C_J, are constant.

Because V is constant, the mass balance for the tank is:

$$\rho \frac{dV}{dt} = q_F - q = 0; \text{ thus } q = q_F$$

Energy balance for tank:

$$\rho V C \frac{dT}{dt} = q_F \rho C (T_F - T) - K q_J^{\,0.8} A (T - T_J) \tag{1}$$

Energy balance for the jacket:

$$\rho_J V_J C_J \frac{dT_J}{dt} = q_J \rho_J C_J (T_i - T_J) + K q_J^{\,0.8} A (T - T_J) \tag{2}$$

where A is the heat transfer area (in ft^2) between the process liquid and the coolant.

Eqs.1 and 2 comprise the dynamic model for the system.

2.9

Assume that the feed contains only A and B, and no C. Component balances for A, B, C over the reactor give.

$$V\frac{dc_A}{dt} = q_i c_{Ai} - q c_A - V k_1 e^{-E_1/RT} c_A \tag{1}$$

$$V\frac{dc_B}{dt} = q_i c_{Bi} - q c_B + V(k_1 e^{-E_1/RT} c_A - k_2 e^{-E_2/RT} c_B) \tag{2}$$

$$V\frac{dc_C}{dt} = -q c_C + V k_2 e^{-E_2/RT} c_B \tag{3}$$

An overall mass balance over the jacket indicates that $q_c = q_{ci}$ because the volume of coolant in jacket and the density of coolant are constant.

Energy balance for the reactor:

$$\frac{d\left[\left(Vc_A M_A S_A + Vc_B M_B S_B + Vc_C M_C S_C\right)T\right]}{dt} = \left(q_i c_{Ai} M_A S_A + q_i c_{Bi} M_B S_B\right)(T_i - T)$$
$$-UA(T - T_c) + (-\Delta H_1)V k_1 e^{-E_1/RT} c_A + (-\Delta H_2)V k_2 e^{-E_2/RT} c_B \tag{4}$$

where M_A, M_B, M_C are molecular weights of A, B, and C, respectively
S_A, S_B, S_C are specific heats of A, B, and C.
U is the overall heat transfer coefficient
A is the surface area of heat transfer

Energy balance for the jacket:

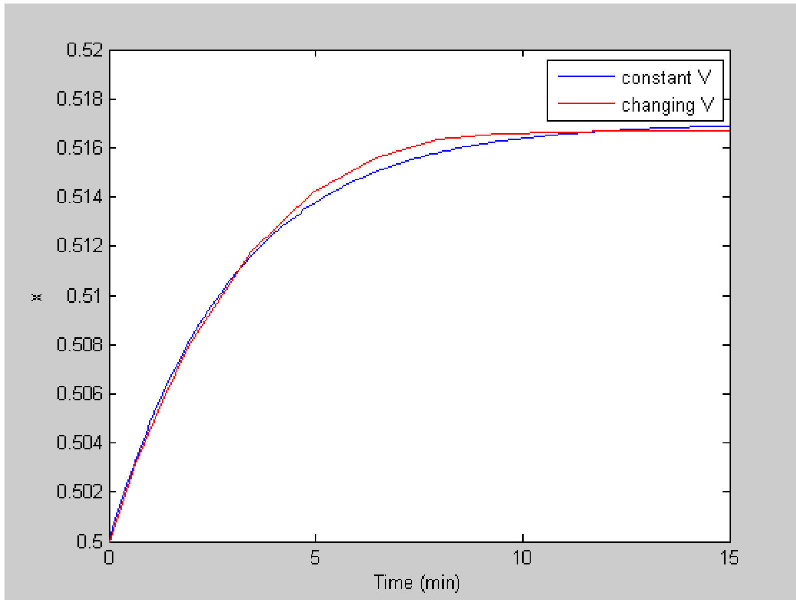
$$\rho_j S_j V_j \frac{dT_c}{dt} = \rho_j S_j q_{ci}(T_{ci} - T_c) + UA(T - T_c) \tag{5}$$

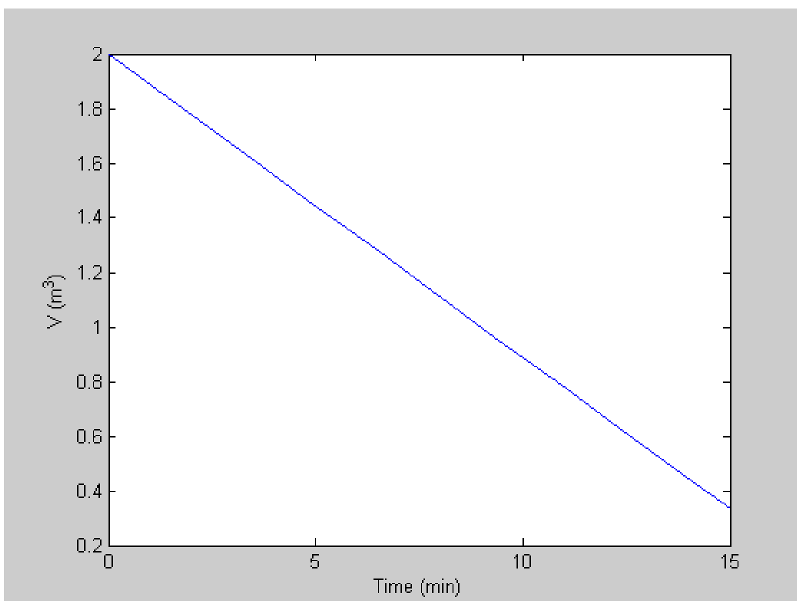
where:
ρ_j, S_j are density and specific heat of the coolant.
V_j is the volume of coolant in the jacket.

Eqs. 1 - 5 represent the dynamic model for the system.

2.10

The plots should look as shown below:





Notice that the functions are only good for t = 0 to t = 18, at which point the tank is completely drained. The concentration function blows up because the volume function is negative.